

Analysing and controlling the tax evasion dynamics via majority-vote model

F.W.S. Lima

Departamento de Física, Universidade Federal do Piauí, 64049-550, Teresina
- PI, Brazil.

e-mail: fwslima@gmail.com, wel@ufpi.edu.br

Abstract: Within the context of agent-based Monte-Carlo simulations, we study the well-known majority-vote model (MVM) with noise applied to tax evasion on simple square lattices, Voronoi-Delaunay random lattices, Barabasi-Albert networks, and Erdős-Rényi random graphs. In the order to analyse and to control the fluctuations for tax evasion in the economics model proposed by Zaklan, MVM is applied in the neighborhood of the noise critical q_c . The Zaklan model had been studied recently using the equilibrium Ising model. Here we show that the Zaklan model is robust and can be reproduced also through the nonequilibrium MVM on various topologies.

Keywords: Opinion dynamics, Sociophysics, Majority vote, Nonequilibrium.

1. Introduction

The Ising model [1, 2] has been used during long time as a "toy model" for diversives objectives, as to test and to improve new algorithms and methods of high precision for calculation of critical exponents in Equilibrium Statistical Mechanics using the Monte Carlo method as Metropolis [3], Swendsen-Wang [4], Wang-Landau [5] algorithms, Single histogram [6] and Broad histogram [7] methods. The Ising model was already applied decades ago to explain how a school of fish aligns into one direction for swimming [8] or how workers decide whether or not to go on strike [9]. In the Latané model of Social Impact [10] the Ising model has been used to give a consensus, a fragmentation into many different opinions, or a leadership effect when a few people change the opinion of lots of others. To some extent the voter model of Liggett [11] is an Ising-type model: opinions follow the majority of the neighbourhood, similar to Schelling [12], all these cited model and others can be found out in [13].

Realistic models of tax evasion appear to be necessary because tax evasion remain to be a major predicament facing governments [14, 15, 16, 17]. Experimental evidence provided by Gächter [18] indeed suggests that tax

payers tend to condition their decision regarding whether to pay taxes or not on the tax evasion decision of the members of their group. Frey and Torgler [19] also provide empirical evidence on the relevance of conditional cooperation for tax morale. Following the same context, recently, Zaklan et al. [20, 21] developed an economics model to study the problem of tax evasion dynamics using the Ising model through Monte-Carlo simulations with the Glauber and heatbath algorithms (that obey detailed balance - equilibrium) to study the proposed model.

G. Grinstein et al. [22] have argued that nonequilibrium stochastic spin systems on regular square lattices with up-down symmetry fall into the universality class of the equilibrium Ising model [22]. This conjecture was confirmed for various Archimedean lattices and in several models that do not obey detailed balance [23, 24, 25, 26, 27]. The majority-vote model (MVM) is a nonequilibrium model proposed by M.J. Oliveira in 1992 and defined by stochastic dynamics with local rules and with up-down symmetry on a regular lattice shows a second-order phase transition with critical exponents β , γ , ν which characterize the system in the vicinity of the phase transition identical [26, 28] with those of the equilibrium Ising model [1] for regular lattices. Lima et al. [29] studied MVM on Voronoi-Delaunay random lattices with periodic boundary conditions. These lattices possess natural quenched disorder in their connections. They showed that presence of quenched connectivity disorder is enough to alter the exponents β/ν and γ/ν from the pure model and therefore that is a relevant term to such non-equilibrium phase-transition with disagree with the arguments of G. Grinstein et al. [22]. Recently, simulations on both *undirected* and *directed* scale-free networks [30, 31, 33, 34, 35, 36, 37], random graphs [38, 39] and social networks [40, 41, 42], have attracted interest of researchers from various areas. These complex networks have been studied extensively by Lima et al. in the context of magnetism (MVM, Ising, and Potts model) [43, 44, 45, 46, 47, 48], econophysics models [21, 49] and sociophysics model [50].

In the present work, we study the behavior of the tax evasion on two-dimensional square lattices using MVM dynamics and furthermore add a policy makers's tax enforcement mechanism consisting of two components: a probability of an audit each person is subject to in every period and a length of time detected tax evaders remain honest. We aim to extend the study of Zaklan et al. [20, 21], which illustrates how different levels of enforcement affect the tax evasion over time, as an alternative model of nonequilibrium to the Ising model that is capable of reproduce the same results for analysis

and control the fluctuation of the tax evasion. This MVM shows that the Zaklan model is very robust for equilibrium and nonequilibrium models and also for various topologies. This does not always happen with other models.

The remainder of our paper is organised as follows. In section 2, we present the Zaklan model evolving with dynamics of MVM. In section 3 we make an analysis of tax evasion dynamics with the Zaklan model on two-dimensional square lattices using MVM for their temporal evolution under different enforcement regimes; we discuss the results obtained. In section 4 we show that the MVM model also is capable to control the different levels of the tax evasion analysed in section 3, as it was made by Zaklan et al. [21] using Ising models. We use the enforcement mechanism cited above on various structures: square lattice, Voronoi-Delaunay random lattice, Barabási-Albert network and Erdős-Rényi graph; we discuss the resulting tax evasion dynamics. Finally in section 5 we present our conclusions about the study of the Zaklan model using MVM.

2. Zaklan model

On a square lattice where each site of the lattice is inhabited, at a time step, by an agent with "voters" or spin variables σ taking the values $+1$ representing an honest tax payer, or -1 trying to at least partially escape her tax duty. Here is assumed that initially everybody is honest. Each period individuals can rethink their behavior and have the opportunity to become the opposite type of agent they were in previous period. In each time period the system evolves by a single spin-flip dynamics with a probability w_i given by

$$w_i(\sigma) = \frac{1}{2} \left[1 - (1 - 2q) \sigma_i S \left(\sum_{\delta=1}^{k_i} \sigma_{i+\delta} \right) \right], \quad (1)$$

where $S(x)$ is the sign ± 1 of x if $x \neq 0$, $S(x) = 0$ if $x = 0$, and the summation runs over all k_i nearest-neighbour sites $\sigma_{i+\delta}$ of σ_i . In this model an agent assumes the value ± 1 depending on the opinion of the majority of its neighbors. The control noise parameter q plays the role of the temperature in equilibrium systems and measures the probability of aligning antiparallel to the majority of neighbors. Then various degrees of homogeneity regarding either position are possible. An extremely homogenous group is entirely made of honest people or tax evaders, depending of the sign $S(x)$ of the majority of neighbors. If $S(x)$ of the neighbors is zero the agent σ_i will be honest or evader in the next time period with probability $1/2$. We further introduce a

probability of an efficient audit (p). Therefore, if tax evasion is detected, the agent must remain honest for a number k of time steps. Here, one time step is one sweep through the entire lattice.

3. Analysing tax evasion dynamics

Here then as Zaklan et al [20] we also use square lattices with lattice size $L = 1000$ ($L \times L = 10^6$ sites) and simulate tax evasion dynamics for various values of q and various punishments (k). When k is zero, no punishment is present and the model describes the baseline majority-vote model. Several degrees of punishment are introduced for different noise (q) or "Social Temperature", by setting k consecutively equal to 10 and 50 periods for all considered levels of the noise. The probability of an audit is sequentially increased, in steps, from 0 to 100 percent. For a given probability of an audit the dynamics of the tax evasion (measured as portion of the entire population) is shown over 300 time steps. Here, we have observed the behavior of tax evasion dynamics on a square lattice for high $q = 0.70$ and low $q = 0.050$ noise values of the critical value for square lattices ($q_c = 0.075$) the MVM model. For both periods time punishment, $K = 50$ and 10, we also use two probability of an audit is either at a realistic level ($p = 0.05$) or at a rather high level ($p = 0.9$) values in this model. The tax evaders have the greatest influence to turn honest citizens into tax evaders if they constitute a majority in given neighbourhood. If the majority evades, one is likely to also evade. On other way, if the majority of agents in the vicinity are honest, the respective individual is likely to become a honest citizen even though it has been a tax evader before. Therefore, in the fig. 1-3, have been showed that on square lattice the MVM model (nonequilibrium model) is capable to reproduce the same behavior and results for analysis of tax evasion levels, as Zaklan et al. [20] made using the Ising model (equilibrium model) on square lattices. How strong the influence from the neighbourhood is can be controlled by adjusting or setting the noise parameter, q , next to (q_c).

— Figure 1 goes about here —

In figure 1 we plot tax evasion for high noise $q = 0.70$, for punishment periods $k = 50$ and 10. If the penalty is high enough, $k = 50$, tax evasion can be reduced to 0% in the short run, given that the probability of an audit p is sufficiently high. In the case of $k = 50$ and $p = 90\%$, picture (a) of Fig. 1, within only a few time steps each individual eventually is

forced to remain honest. This happens, because the spins flip more often at this noise, which is too high compared to the critical noise of this model in the square lattice ($q_c = 0.075$) [26], where q_c is the critical noise for phase transition of MVM on square lattice. In this picture (a) we observe peaks of different levels of the tax evasion, that result from the fact that 90% of the initial large number of tax evaders gets caught and after k iterations becomes free to decide whether to evade or not. Moreover consecutive peaks in non-compliance diminish less over time and it takes longer until perfect non-compliance is established, further out on the time scale at which the evolution of tax evasion is considered. Therefore, in our simulation using MVM the perfect compliance is not attained in agreement with the results observed for Zaklan et al. [20] using the Ising model where an equilibrium level of about 2% non-compliance is attained.

If punishment is set equal to $k = 10$ periods (picture (b) of Fig. 1) tax evasion reaches 0% in the first ten time steps and finally comes to rest at a level of 10%. When the probability of an audit $p = 0.05$ is much lower the tax evasion level reaches values for non-compliance of about 24% and 43% in pictures (c) and (d), respectively, of Fig. 1. These two pictures show well that punishment is a suitable enforcement mechanism when the probability of an audit is set to a realistic level.

— Figure 2 and 3 goes about here —

We also consider two other noises above q_c , $q = 0.090$, and below it, $q = 0.050$. At $q = 0.090$ the individual becomes tax evader more slowly, because the tax evasion problem is less pronounced already from the beginning compared to higher noises, because spins flip less frequently at lower noises, the same enforcement mechanisms may work less efficiently in the short run than at higher noise. Therefore as shown in Fig. 2 at $q = 0.090$ with $k = 50$ and $p = 0.9$ one clearly sees that evasion cannot be reduced to zero percent any more. At $q = 0.050$ the shape of tax evasion is similar to $q = 0.090$, but the level of tax evasion is smaller, because spins flip less frequently at lower noises $q = 0.050$ as shown in Fig. 3, again enforcement mechanisms cannot work efficiently in the short run as opposed to higher noise ($q = 0.70$). At such low noise individuals seldomly decide to become non-compliant because their vicinity which is mostly compliant on average exerts strong influence on them to be honest as well. Therefore the equilibrium level of tax evasion is smaller than at higher noises.

4. Controlling the tax evasion dynamics

Here, we first will present the baseline case $k = 0$, i.e., no use of enforcement, for different network structure. We use for square lattice $q = 0.95q_c$, for Voronoi-Delaunay lattice at $q = q_c = 0.117$ (critical noise for MVM model on Voronoi-Delaunay lattice), for Barabási-Albert $q = 0.9q_c = 0.306$ and Erdős-Rényi $q = 0.9q_c = 0.181$. All simulation are performed over 20,000 time steps, it is showed in the fig. 4.

For very low noises the part of autonomous decisions almost completely disappears. The individuals then base their decision solely on what most of their neighbours do. A rising noise has the opposite effect. Individuals then decide more autonomously. For the MVM model it is known that for $q > q_c$, half of the people are honest and other half cheat, while for $q < q_c$ states correlated on cheating or compliance prevail for most of the time. Because this behavior we set some values closed to q_c , where the case that agents distribute in equal proportions onto the two alternatives is excluded. Then have been setting the noise parameter, q , closed to ($q_c = 0.075$) on square lattice, as suggested in the section 3, and ranging the degrees of punishment ($k = 1, 10$ and 50) and audit probability rate ($p = 0.5\%$, 10% and 90%). Therefore, if tax evasion is detected, the enforcement mechanism (p) and the period time of punishment k are triggered in order of to control the tax evasion level. The individual remain honests for a certain number of periods, as explained before in section 2 and 3. We also extend our study to other networks as the Voronoi-Delaunay lattices, Barabási-Albert networks and Erdős-Rényi graphs with $N = 400$ sites. As before the initial configurations is with all honest agents ($\sigma_i = +1$) at fixed "Social Temperature" (q). Here, have been performed simulations of 10,000 to 20,000 time steps.

— Figure 4 goes about here —

In Fig. 4 we plot the baseline case $k = 0$, i.e., no use of enforcement, for the square lattice (a), Voronoi-Delaunay (b), Barabási-Albert (c), and Erdős-Rényi (d) for dynamics of the tax evasion over 20,000 time steps. Although everybody is honest initially, it is impossible to predict which level of tax compliance will be reached at some time step in the future.

— Figure 5 goes about here —

Figure 5 illustrates different simulation settings for square lattice, for each considered combination of degree of punishment ($k = 1, 10$ and 50)

and audit probability rate ($p = 0.5\%$, 10% and 90%), where the tax evasion is plotted over 10,000 time steps. Here we show that even a very small level the enforcement ($p = 0.5\%$ and $k = 1$) suffices to reduce fluctuations in tax evasion and to establish mainly compliance. Both a rise in audit probability (greater p) and higher penalty (greater k) work to flatten the time series of tax evasion and to shift the band of possible non-compliance values towards more compliance. However, the simulations show that even extreme enforcement measures ($p = 90\%$ and $k = 50$) cannot fully solve the problem of tax evasion.

— Figure 6 goes about here —

In Fig. 6 we display tax evasion for Voronoi-Delaunay random lattice, Barabási-Albert network, and Erdős-Rényi random graph for different enforcement for $k = 1, 10$, and 50 with the same audit probability $p = 1\%$. We observe for the Voronoi-Delaunay random lattice and Barabási-Albert network that the tax evasion level decreases with increasing time periods k of punishment. For the case of the Erdős-Rényi random graph the tax evasion fluctuations for $k = 1$ and $p = 1\%$, case (c), is identical to the baseline case for this random graph, only for $k = 10$ and 50 at $p = 1\%$ the tax evasion level decreases to about 48% (f) and 40% (i), showing that the control for tax evasion for small audit probability $p = 1\%$ is more difficult to reach in these Erdős-Rényi random graphs.

— Figure 7 goes about here —

In Fig. 7 we plot tax evasion for Barabási-Albert networks and Erdős-Rényi random graphs, again for different enforcement $k = 1, 10$, and 50 , but now with audit probability $p = 4.5\%$. For Barabási-Albert networks the tax evasion level decreases with increasing audit probability p showing that an increase of the audit probability favors the control of tax evasion. Again, for $k = 1$ and $p = 4.5\%$ (d) the tax evasion for the Erdős-Rényi graphs is identical for case baseline and the case (c) of Fig. 6, showing that for these random graphs the time period k of punishment is important to control tax evasion.

5. Conclusion

In summary, tax evasion can vary widely across nations, reaching extremely high values in some developing countries. Wintrobe and Gërkhani

[51] explains the observed higher level of tax evasion in generally less developed countries with a lower amount of trust that people have in governmental institutions. In this work we show that Zaklan's model is very robust for analysis and control of tax evasion, using dynamics with equilibrium [21], Ising model, and also nonequilibrium, MVM. The Zaklan model is also found to be robust in various topologies used here.

The author thank Dietrich Stauffer for the revision of this paper. We also acknowledge the Brazilian agency FAPEPI (Teresina-Piauí-Brasil) for its financial support. This work also was supported the system SGI Altix 1350 the computational park CENAPAD.UNICAMP-USP, SP-BRAZIL.

References

- [1] W. Lenz, Z. Phys. 21, 613 (1921). E. Ising, Z. Phys. 31,235 (1925).
- [2] R.J. Baxter, Exactly solved models in statistical mechanics, London, Academic Press, 1982.
- [3] N. Metropolis, A. W. Rosenbluth, M. N. Rosenbluth, A. H. Teller, E. Teller, J. Chem. Phys. 21, 1087 (1953).
- [4] R. H. Swendsen, J.-S. Wang, Phys. Rev. Lett. 58, 86 (1987).
- [5] F. Wang, D. P. Landau, Phys. Rev. Lett. 86, 2050 (2001).
- [6] A. M. Ferrenberg, R. H. Swendsen, Phys. Rev. Lett., 61, 2635 (1988).
- [7] P. M. C. de Oliveira, T. J. P. Penna, Hans J. Herrmann, Braz J. Phys. 26, 677 (1996).
- [8] E. Callen, D. Shapero, Physics Today 27 (2), 23 (1974).
- [9] S. Galam, Y. Gefen, Y. Shapir, J. Mathematical Sociology 9, 1 (1982).
- [10] B. Latané, Am. Psychologist 36, 343 (1981).
- [11] T. M. Liggett, Interacting Particles Systems, Springer, New York (1985).
- [12] T.C. Schelling, J. Mathematical Sociology 1, 143 (1971).

- [13] D. Stauffer, S. Moss de Oliveira, P.M.C. de Oliveira, J.S. Sá Martins, "Biology, Sociology, Geology by Computational Physicists", Elsevier, Amsterdam (2006).
- [14] K. Bloomquist, Social Science Computer Review 24, 411 (2006).
- [15] J. Andreoni, B. Erard, J. Feinstein, Journal of Economic Literature 36, 818-860 (1998).
- [16] L. Lederman, Public Law Research paper No. 50, (2003).
- [17] J. Slemrod, Journal of Economic Perspective 21, 25(2007).
- [18] S. Gächter, Discussion Papers 2006-03, CeDEx, University of Nottingham, 2006.
- [19] B.S. Frey, B. Togler, IEW-Working papers 286, Institute for Empirical Research in Economics, University of Zurich, (2006).
- [20] G. Zaklan, F. Westerhoff, D. Stauffer, arXiv:0801.2980. J. Econ. Interact. Coordination.
- [21] G. Zaklan, F.W.S. Lima, F. Westerhoff, Physica A 387, 5857 (2008).
- [22] G. Grinstein, C. Jayaprakash, Yu He, Phys. Rev. Lett. 55, 2527 (1985).
- [23] C.H. Bennett, Phys. Rev. Lett. 55, 657 (1985).
- [24] J.S. Wang and J.L. Lebowitz, J. Stat. Phys. 51, 893 (1988).
- [25] M.C. Marques, Phys. Lett. A 145, 379 (1990).
- [26] M.J. Oliveira, J. Stat. Phys. 66, 273 (1992).
- [27] F.W.S. Lima, Int. J. Mod. Phys. C 17, 1273 (2006).
- [28] M. A. Santos, S. Teixeira, J. Stat. Phys. 78 , 963 (1995).
- [29] F.W.S. Lima, U.L. Fulco, R.N. Costa Filho, Phys. Rev. E 71, 036105 (2005).
- [30] M.E.J. Newman, S.H. Strogatz, D.J. Watts, Phys. Rev. E 64 026118 (2001) .

- [31] A.D. Sanchez, J.M. Lopez, M.A. Rodriguez, Phys. Rev. Lett. 88 048701 (2002).
- [32] G. Palla, I.J. Farkas, P. Pollner, I. Derenyi, T. Vicsek, New J. Phys. 9, 186 (2007).
- [33] A.-L. Barabási, R. Albert, Science 286, 509 (1999).
- [34] A. Aleksiejuk, J.A. Holyst, D. Stauffer, Physica A 310 (2002) 269.
- [35] M.A. Sumour, M.M. Shabat, Int. J. Mod. Phys. C 16, 585 (2005).
- [36] M.A. Sumour, M.M. Shabat, D. Stauffer, Islamic University Journal (Gaza) 14, 209 (2006). cond-mat/0504460/at www.arXiv.org.
- [37] F.W.S. Lima, D. Stauffer, Physica A 359, 423 (2006).
- [38] P. Erdős and A. Rényi, Publ. Math. Debrecen 6, 290 (1959); P. Erdős, A. Rényi, Publ. Math. Inst. Hung. Acad. Sci. 5 , 17 (1960); P. Erdős and A. Rényi, Bull. Inst. Int. Stat. 38 , 343 (1961).
- [39] B. Balobás, Random Graphs, Academic Press (New York) (1985).
- [40] D.J. Watts, S. H Strogatz, Nature 393, 440 (1998).
- [41] S. Wasseman, K. Faust, B. Balobás, Social Networks Analysis, Cambridge University Press. (Cambridge) (1994).
- [42] D. Stauffer, M. Hohnisch, S. Pittnauer, Physica A 370, 734 (2006).
- [43] F.W.S. Lima, Int. J. Mod. Phys. C. 17, 1267 (2006).
- [44] F.W.S. Lima, Int. J. Mod. Phys. C 17, 1257 (2006).
- [45] F.W.S. Lima, Commun. Comput. Phys. 2, 358 (2007).
- [46] F.W.S. Lima, Commun. Comput Phys. 2, 522 (2007).
- [47] F.W.S. Lima, Int. J. Mod. Phys. C 19, 1063 (2008).
- [48] F.P. Fernandes, F.W.S. Lima, Int. J. Mod. Phys. C 19, 1777 (2008).
- [49] F.W.S. Lima, G. Zaklan, Int. J. Mod Phys. C 19, 1797 (2008).

- [50] F.W.S. Lima, T. Hadzibeganovic, D. Stauffer, *Physica A* 388, 4999 (2009).
- [51] R. Wintrobe, K. Gërxhani, *Proceedings of the Annual Meeting of the European Public Choice Society 2004*, (2004).

Noise , $q = 0.70$

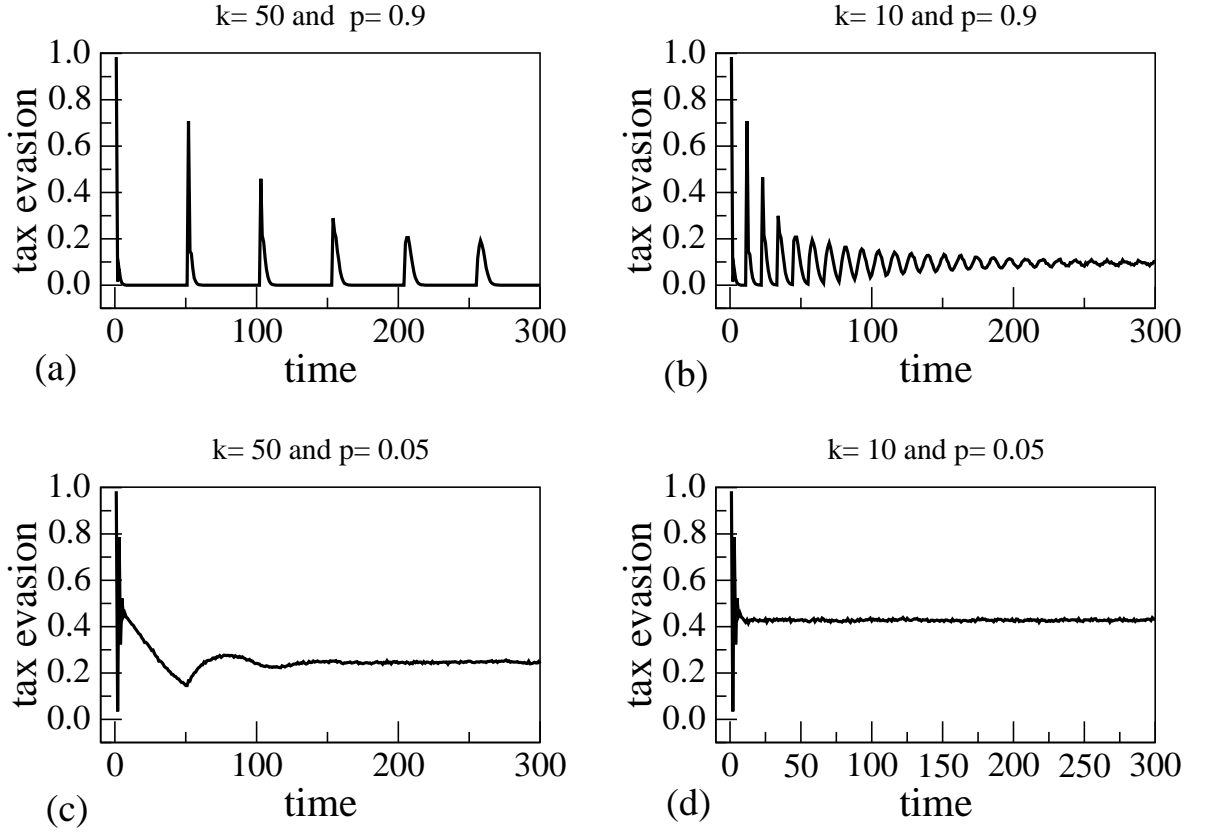


Figure 1: Tax evasion dynamics for high "social temperature" $q = 0.70$ ($q_c = 0.075$) and 300 time step, $L = 1000$ (10^6 sites), with length of punishment at either $k = 50$ or 10 periods and two different probabilities of an audit, i.e. $p = 0.05$ and $p = 0.9$.

Noise , $q = 0.09$

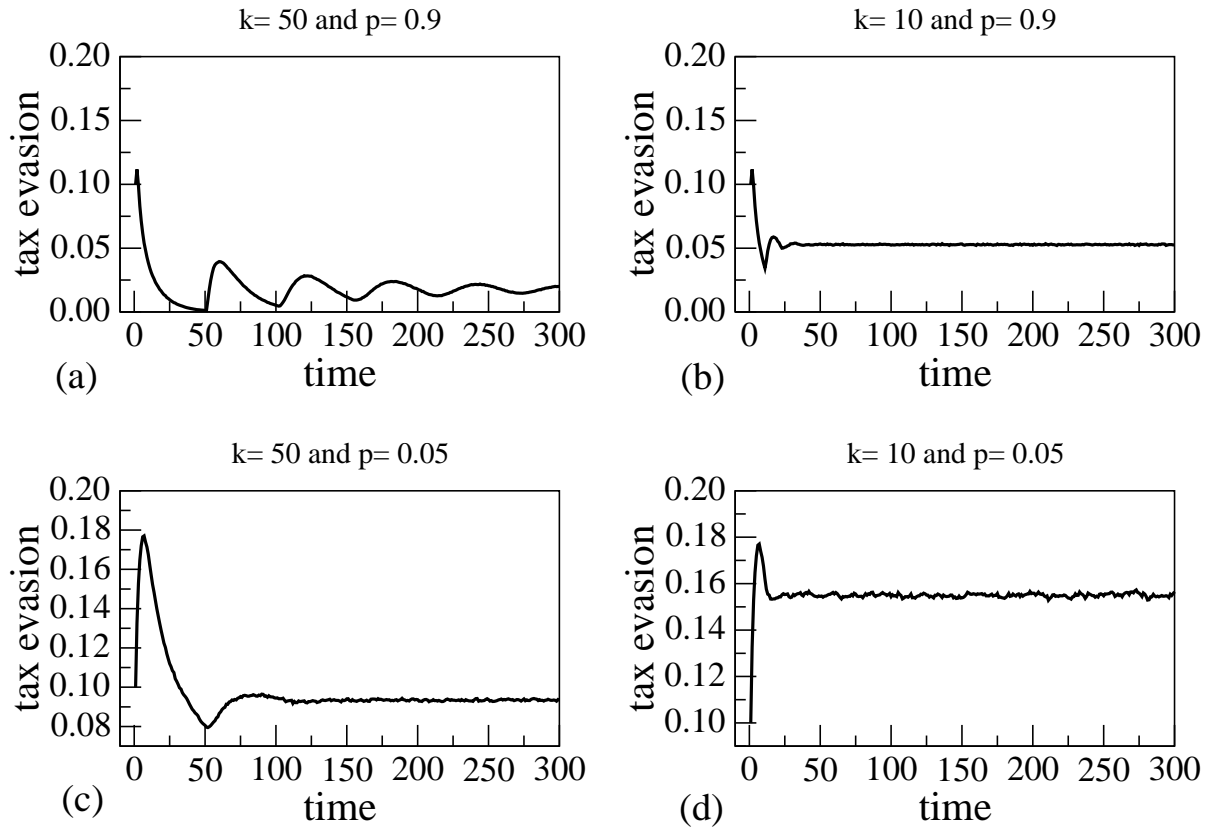


Figure 2: Tax evasion dynamics for "social temperature" $q = 0.090$ ($q_c = 0.075$). The same simulation design as in figure 1.

Noise , $q = 0.05$

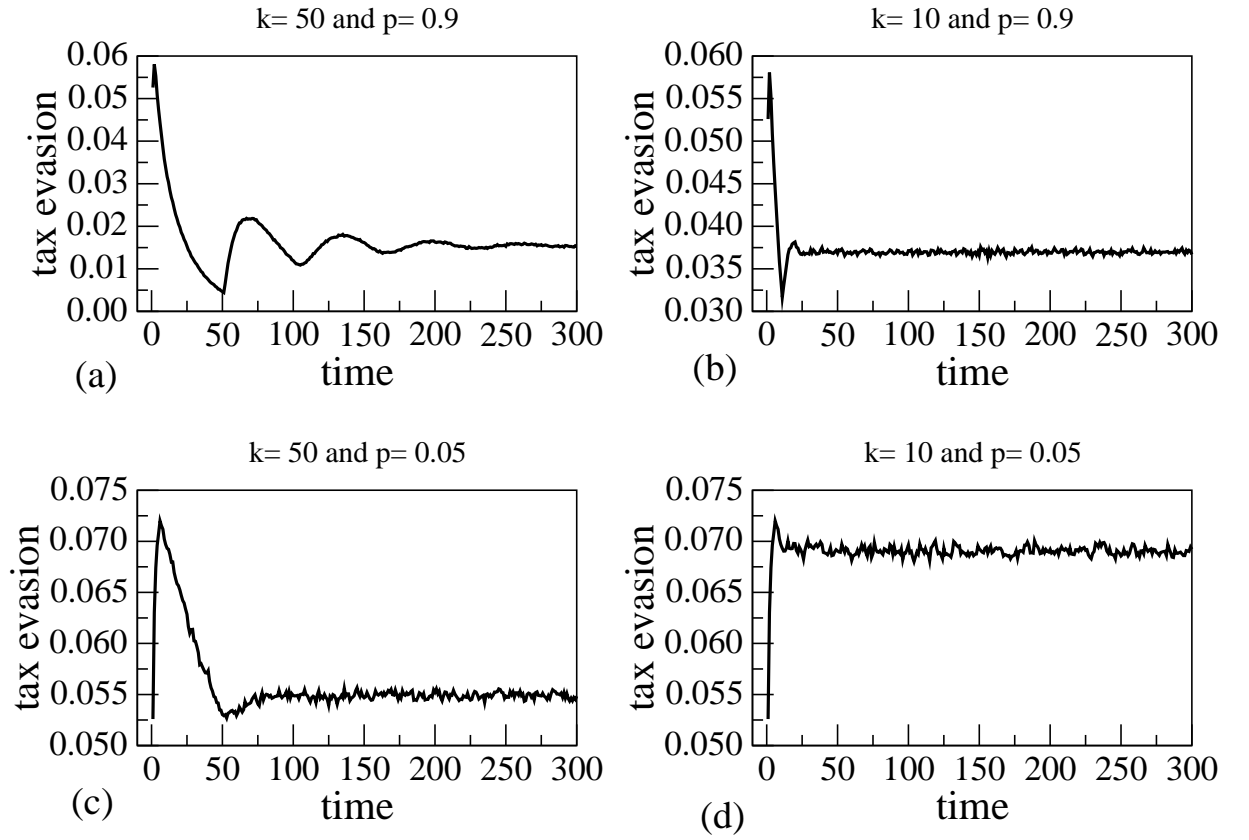


Figure 3: Tax evasion dynamics for "social temprature" $q = 0.050$. The same simulation design as in figure 1.

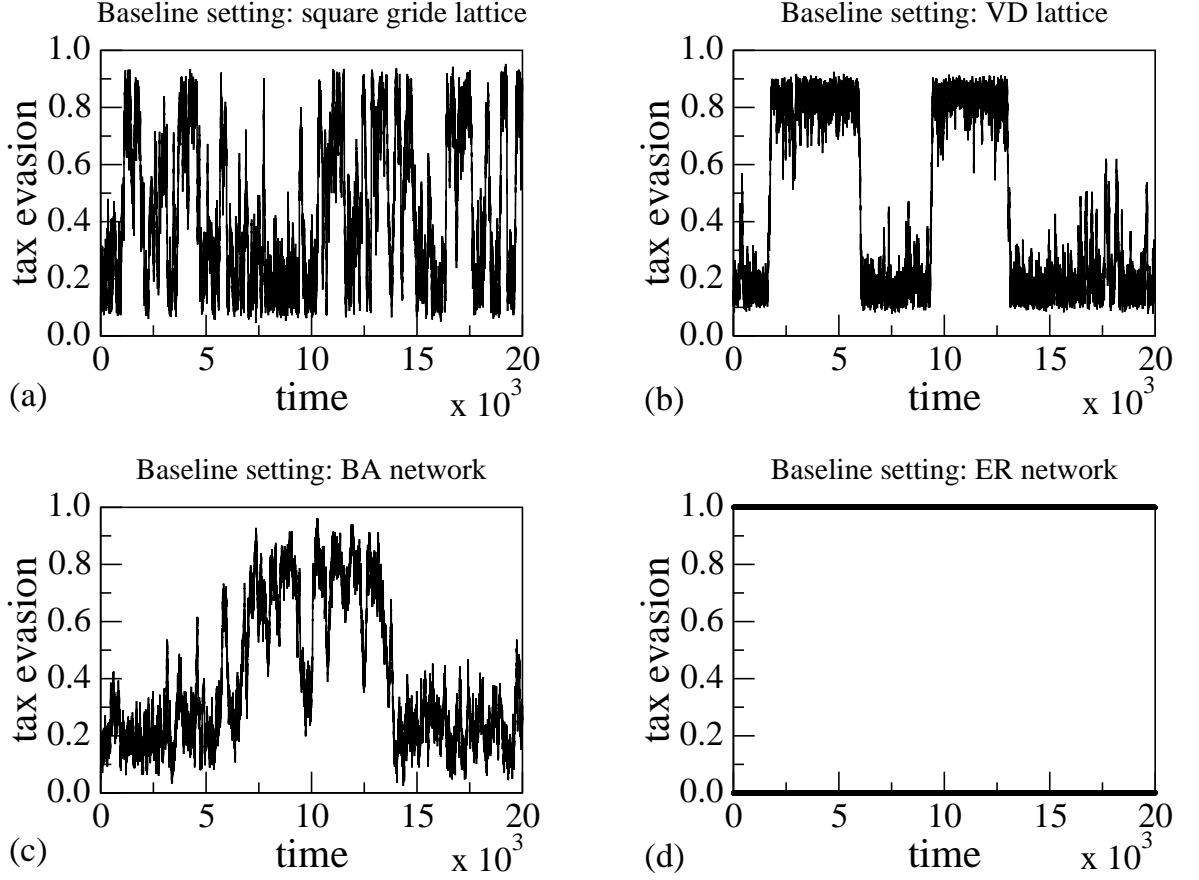


Figure 4: Baseline case for different network structure. Where we use for square lattice $q = 0.95q_c$, for Voronoi-Delaunay lattice at $q = q_c = 0.117$ (critical noise for MVM model on Voronoi-Delaunay lattice), for Barabási-Albert $q = 0.9q_c = 0.306$ and Erdős-Rényi $q = 0.9q_c = 0.181$. All simulation are performed over 20,000 time steps.

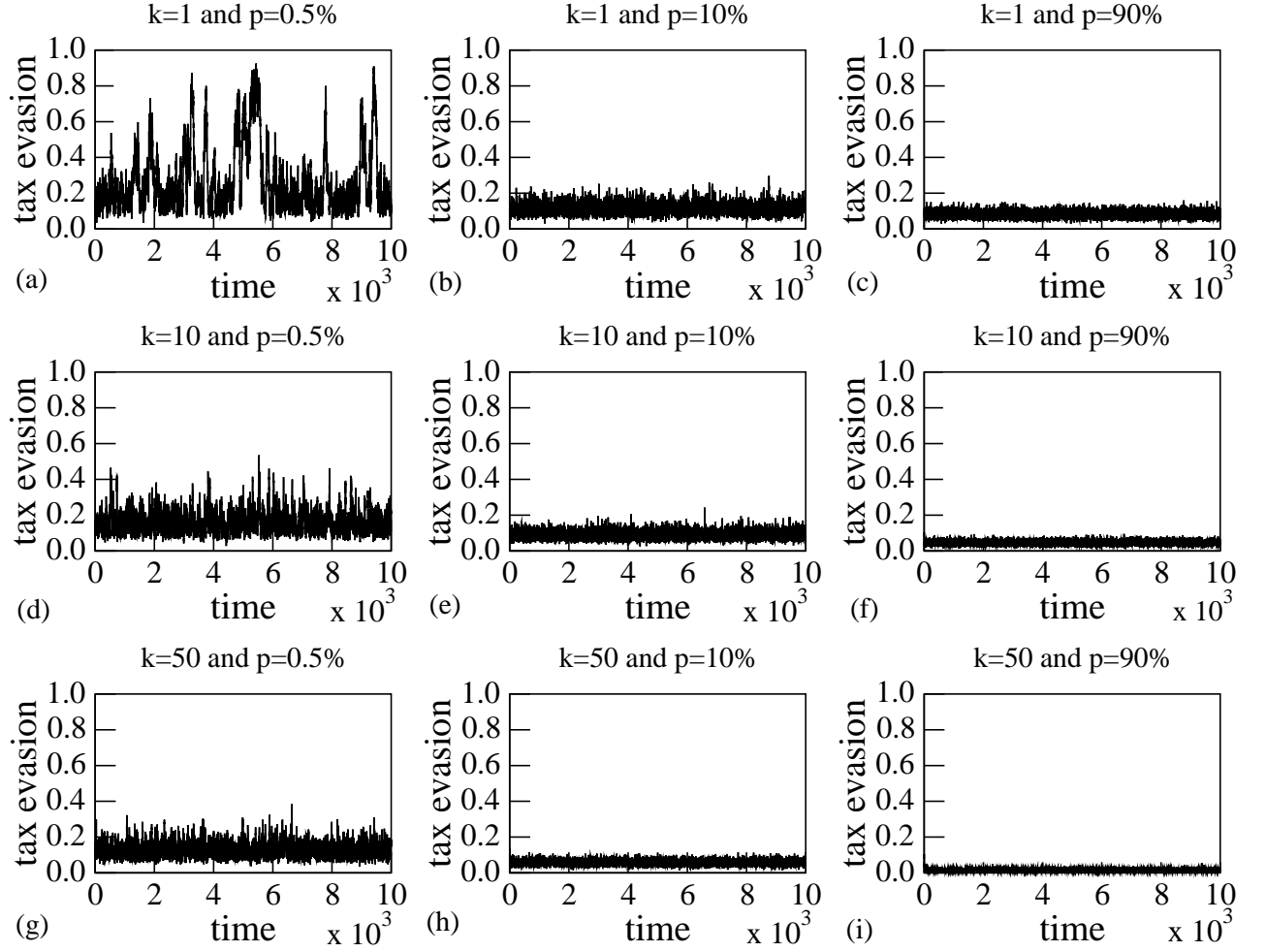


Figure 5: The square lattice model of tax evasion with various degrees p of enforcement. $q = 0.95q_c$ and 20,000 time steps.

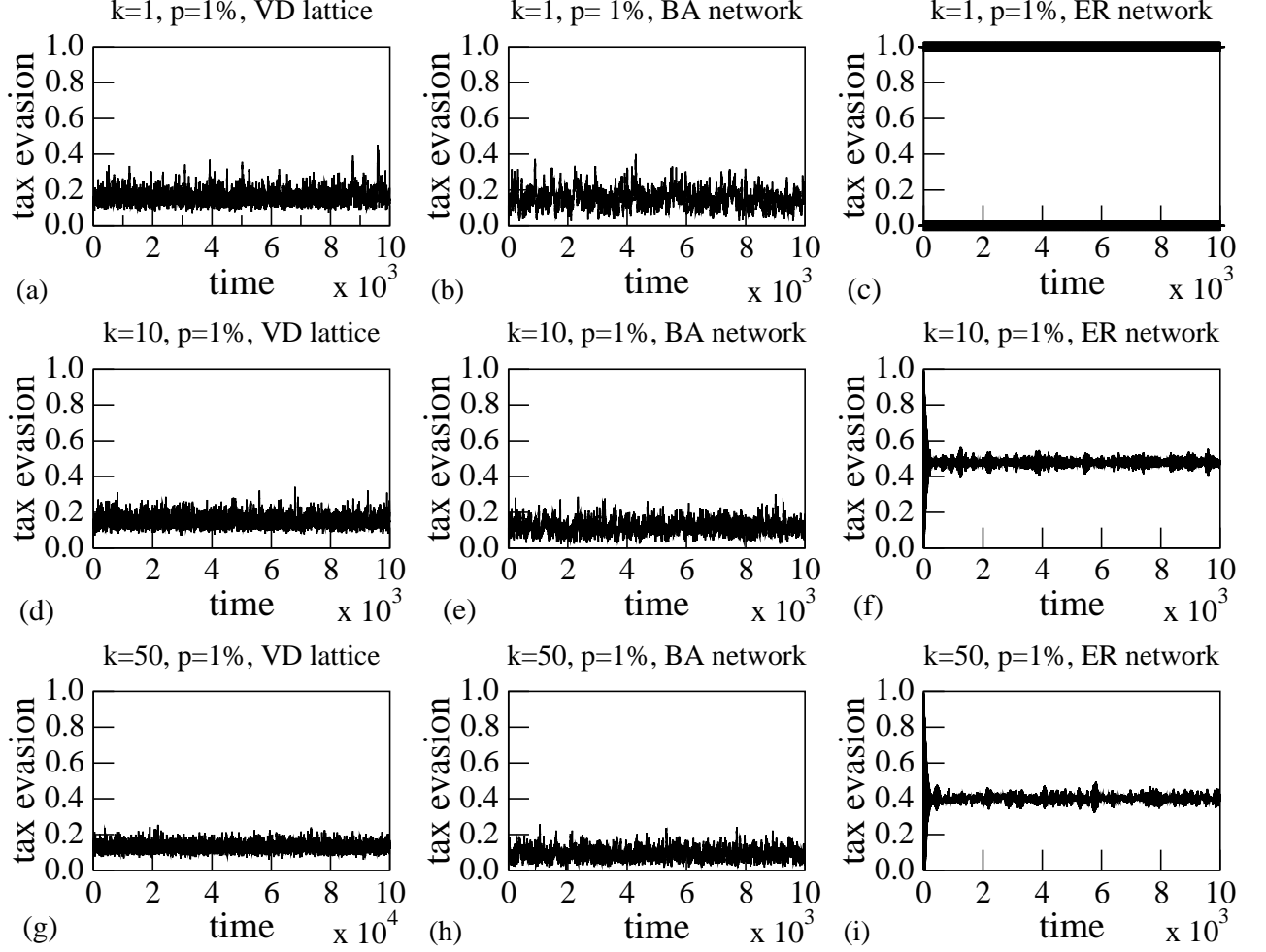


Figure 6: The first column displays tax evasion for different enforcement regimes for Voronoi-Delaunay network ((a), (d), and (g)). The next two column depict the tax evasion for Barabási-Albert network ((b), (e), and (h)) and Erdős-Rényi graph((c), (f), and (i)). Again, we use 10,000 time steps.

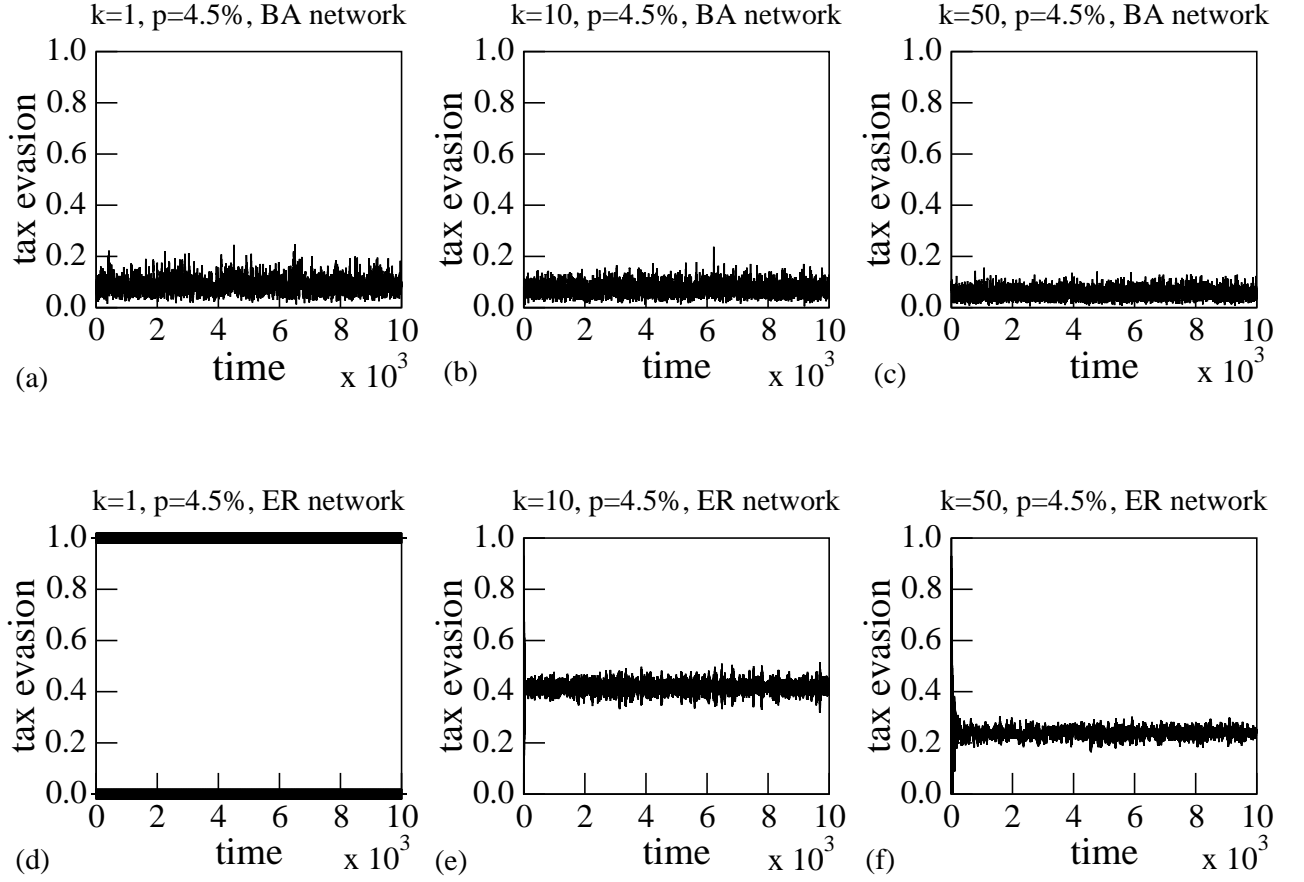


Figure 7: The first line display the resulting tax evasion for different enforcement regimes for Barabási-Albert networks ((a), (b), and (c)) for degrees of punishment $k = 1, 10$ and 50 and audit probability $p = 4.5\%$. The second line depicts tax evasion for Erdős-Rényi graphs((d), (e), and (f)) for the same values the case as line 1. Again, we use 10,000 time steps.